3.5 Vector Equation of Curves

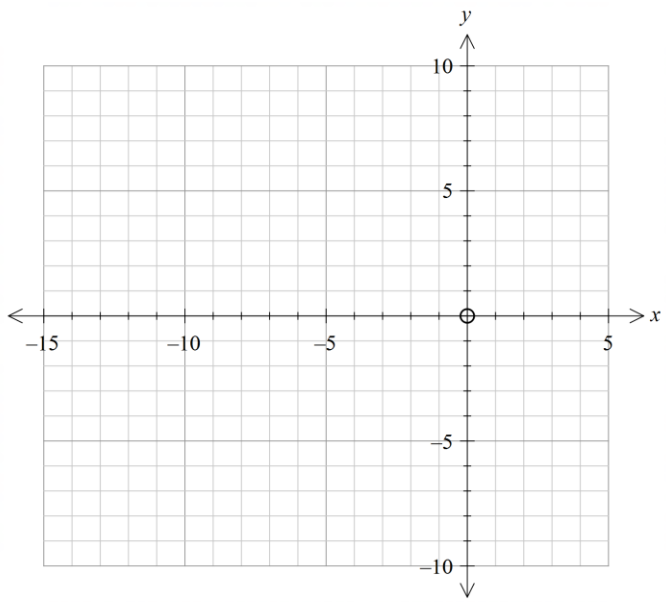
Problems Worksheet

$\pi \int_a^b y^2 \, dx$
Step Up In Mathematics Specialist Unit 3 and 4

1. Consider a body with its position described by the following vector equation, where t is in seconds and $\boldsymbol{r}$ in metres.
   $\boldsymbol{r} = (3 - t^2)\boldsymbol{i} + (t + 3)\boldsymbol{j}$

   a. For $\{t \in \mathbb{Z}: 0 \leq t \leq 4\}$, create a table detailing the position of the particle for time t.

   b. Sketch the path of the body on the following axes. Indicate clearly its direction of travel.



2. Convert the following to Cartesian form.

   a. $\boldsymbol{r} = (t - 1)\boldsymbol{i} + \left(\sqrt{t + 4}\right)\boldsymbol{j}$

   b. $\boldsymbol{r} = (2t^2 - 5)\boldsymbol{i} + \left(\sqrt{t}\right)\boldsymbol{j}$

   c. $\boldsymbol{r} = (8\sin t \cos t)\boldsymbol{i} + (8\cos^2 t - 4)\boldsymbol{j}$

   d. $\boldsymbol{r} = (8\sin t \cos t)\boldsymbol{i} + (6\cos^2 t - 1)\boldsymbol{j}$

   e. $\boldsymbol{r} = (3\sin 2t)\boldsymbol{i} + (\cos^2 t - \sin^2 t)\boldsymbol{j}$

3. Write each of the following Cartesian equations of curves as an equivalent vector equation. From the Cartesian equation of each curve, determine a second vector equation which is also valid.

   a. $y = 9x + 7$

   b. $y = 2x + 3$

4. Determine the intersection of the following spheres and lines, if they exist.

a. $L_1 : \boldsymbol{r} = \begin{pmatrix} 25 \\ 10 \\ 8 \end{pmatrix} + \lambda \begin{pmatrix} 8 \\ 3 \\ 1 \end{pmatrix}$. and $\left| \boldsymbol{r} - \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} \right| = \sqrt{65}$

b. $L_2 : \boldsymbol{r} = \begin{pmatrix} -1 \\ 4 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$. and $\left| \boldsymbol{r} - \begin{pmatrix} -5 \\ 6 \\ 5 \end{pmatrix} \right| = \sqrt{141}$

c. $L_3 : \boldsymbol{r} = \begin{pmatrix} -4 \\ 2 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$. and $\left| \boldsymbol{r} - \begin{pmatrix} 0 \\ -3 \\ 3 \end{pmatrix} \right| = \sqrt{3}$

5. Determine the value(s) of k such that there will be one intersection between the line and sphere below.

$$\boldsymbol{r} = \begin{pmatrix} -3 \\ -6 \\ 11 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \text{ and } \left| \boldsymbol{r} - \begin{pmatrix} 1 \\ 6 \\ k \end{pmatrix} \right| = 4\sqrt{5}.$$

6. Consider the sphere centred at the origin with radius 8 units, and the line $\boldsymbol{r} = \begin{pmatrix} 10\sqrt{2} + 2\sqrt{2}\lambda \\ 28\sqrt{3} + 4\sqrt{3}\lambda \\ -10\sqrt{2} - 2\sqrt{2}\lambda \end{pmatrix}$. The segment of the line which intersects with and is inside the sphere represents one side of a regular polygon circumscribed inside the sphere. The two intersections are adjacent vertices of the polygon. Determine the number of sides of the regular polygon.